Lecture 10 – Populations of Neurons: Fokker-Planck equation

Course Neural Networks and Biological Modeling

- Review of Integrate-and-Fire
- Definition of homogeneous Population
- Density of membrane potentials
- Flux
- Fokker-Planck Equation
- Threshold and Reset

BOOK: Spiking Neuron Models, W. Gerstner and W. Kistler
Cambridge University Press, 2002

Chapter 6

Course: Neural Networks and Biological Modeling
Wulfram Gerstner, EPFL
From single Neurons to Populations

Receptive fields, see lecture 4

Visual cortex

Orientation selective

Neighboring neurons have similar properties

From single neurons to populations

10 000 neurons
3 km wires

Random firing in a population of neurons

Input
- low rate
- high rate

Population
- 50 000 neurons
- 20 percent inhibitory
- randomly connected

Diffusive noise (stochastic spike arrival)

Population of spiking neurons

Homogeneous network:
- each neuron receives input from k neurons in network
- each neuron receives the same (mean) external input

Population activity

\[ A(t) = \frac{n(t; t + \Delta t)}{N \Delta t} \]

Stochastic spike arrival:
- excitation, total rate \( R_e \)
- inhibition, total rate \( R_i \)

For any arbitrary neuron in the population

\[ \tau \frac{du}{dt} = -u + R_e \sum_{k} q_k \delta(t - t_k) - R_i \sum_{k} q_k \delta(t - t_k') \]
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Exercise 1: flux caused by stochastic spike arrival

Membrane potential density

Reference level \( u_0 \)

a) Jump at time \( t \)

b) spike arrival rate \( v \)

c) spike arrival rate \( \sum v_i \)

What is the flux across \( u_0 \)?

Blackboard: Slope and density of potentials

Next lecture: 10h15

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Wulfram Gerstner, EPFL
Diffusive noise (stochastic spike arrival)

For any arbitrary neuron in the population
\[
\frac{d}{dt} u = -u + \frac{q}{\tau} \sum_{\text{EPSC}} \delta(t-t_i') - \frac{q}{\tau} \sum_{\text{IPSC}} \delta(t-t_i') + \frac{1}{C} I^\text{ext}(t)
\]

Continuity equation:
\[
\frac{\partial}{\partial t} p(u,t) = -\frac{\partial}{\partial u} J(u,t)
\]
   Flux: - jump (spike arrival)
   - drift (slope of trajectory)

Exercise 2: solution of free Fokker-Planck equation

Membrane potential density: Gaussian

Next lecture: 11h15

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Wulfram Gerstner, EPFL
Population firing rate $A(t) = f$ = single neuron firing rate

Synaptic current pulses
\[
\tau \frac{d}{dt} u = -(u - u_{\text{eq}}) + \sum_{k} q_k \delta(t - t^{'k}) + \sum_{j} q_j \delta(t - t^{''j}) \\
\tau \frac{d}{dt} I = -(I - I_{\text{eq}}) + R I_{\text{epsc}}(t) + \xi(t) + R I_{\text{ipsc}}(t) \\
l(t) = [I_e + I_{\text{noise}}]
\]
effective noise current

Exercise 3: Diffusive noise + Threshold

Membrane potential density
\[
\tau \frac{\partial}{\partial t} p(u, t) = -\frac{\partial}{\partial u} [\gamma(u) p(u, t)] + \frac{1}{2} \sigma^2 \frac{\partial^2}{\partial u^2} p(u, t) + \text{source}
\]

- Calculate distribution $p(u)$
- Determine population firing rate $A$

Miniproject: 12h00

The End