Lecture 12
Course: Neural Networks and Biological Modeling

Connected Populations: oscillations, competition and spatial continuum (field equations)

Population of neurons

\[ A(t) = \frac{n(t; t + \Delta t)}{N\Delta t} \]

\[ \tau \frac{d}{dt} A(t) = -A(t) + g(h(t)) \]

\[ A(t) = g(h(t)) = g(\int k(s)I(t-s)ds) \]

\[ A(t) = g(I(t)) \]

\[ A(t) = g(I(t), I'(t)) \]

Signal transmission in populations of neurons

Population
- 50 000 neurons
- 20 percent inhibitory
- randomly connected

Connections
4000 external
4000 within excitatory
1000 within inhibitory

Input
low rate
high rate

Theory of transients

noise model A
(escape noise/fast noise)

low noise

high noise

Fast transient
\[ A(t) = g(I(t)) \]

Slow transient
\[ A(t) = g(h(t)) \]

But transient oscillations

High-noise activity equation

Population activity

\[ A(t) = g(h(t)) \]

Membrane potential caused by input

\[ \frac{d}{dt} h(t) = -h(t) + R I(t) \]

Slow transient
\[ A(t) = g(h(t)) \]
Full connectivity

\[ \tau \frac{d}{dt} h(t) = -h(t) + RI(t) \]

All neurons receive the same total input current ('mean field')

\[ I_i(t) = J_0 \int_0^\infty \alpha(t-s) \gamma(s) + I^{ext}(t) \]

Index \( i \) disappears

\[ I_i(t) = \frac{1}{N} \sum_j \sum_w w_j \alpha(t^j) \gamma(t^j) + I^{ext}(t) \]

All spikes, all neurons

Exercises 1, now

Exercise 1: homogeneous stationary solution

Next lecture 10:15

Homogeneous network
All neurons are identical,
Single neuron rate = population rate

\[ \nu = A \]

High-noise activity equation

What is this function \( g \)?

\( A_0 = g(h_0) \)

Population activity

\[ A(t) = g(h(t)) \]

Membrane potential caused by input

\[ \tau \frac{d}{dt} h(t) = -h(t) + RI(t) \]

Note: total membrane potential

\( u(t) = h(t) + \eta(t - \hat{t}) \)

Effect of last spike

Spike Response Model

Spike emission

Last spike of \( i \)

All spikes, all neurons

\[ \tau \frac{d}{dt} h(t) = -h(t) + RI(t) \]

Spike reception: EPSP

Spike reception: AP

Firing:

\[ i_\text{firing} = \hat{t} \]
Fully connected network

Spike emission: \( AP = \eta \left( I_i \right)_t \)

\[
\begin{align*}
I(t) &= I^{(st)}(t) + I^{(ext)}(t) \\
I^{(ext)}(t) &= \sum_{j=0}^{N} W_{ij} \left( g(x) \right)_{ij} \\
I^{(ext)}(t) &= \int_0^t \left( \alpha(s) A(t-s) ds \right)
\end{align*}
\]

Synaptic coupling: \( W_{ij} = \frac{J_0}{N} \)

Stability of Asynchronous State

\[ A(t) = \int_{-\infty}^{t} P_{ij}(\tau) \Delta A(t-\tau) d\tau \]

\[ \Delta A(t) = \int_{-\infty}^{t} P_{ij}(\tau) \Delta A(t-\tau) d\tau + A_0 \frac{d}{dt} \int_{-\infty}^{t} I(x) \Delta A(t-x) dx \]

Oscillations

Stationary State/Asynchronous State

\[ u = \left( \frac{C}{\eta} \right) \left( \int_{-\infty}^{t} \Delta A(t-\tau) d\tau \right) \]

Typical mean field (Curie Weiss)

\[ \nu = \frac{1}{T} \int_0^T \left( I_s + \frac{1}{\lambda} \int_0^t \left( \alpha(s) A(t-s) ds \right) \right) = g(h_0) \]

Noise model A (escape noise/fast noise)

Population activity

\[ A(t) = g(h(t)) \]

Membrane potential caused by input

\[ I(t) = I^{(st)}(t) + I^{(ext)}(t) \]

\[ I(t) = I^{(ext)}(t) + J_0 q \left( h(t) \right) \]

\[ I(t) = I^{(ext)}(t) + J_0 q \left( g(h(t)) \right) \]

\[ \frac{d}{dt} h(t) = -h(t) + R I(t) \]

\[ h(t) = h_0 + \Delta h(t) \]

\[ \Delta h(t) = J \int_0^t \left( \alpha(s) A(t-s) ds \right) \]

Search for bifurcation points

\[ \Delta = 0 \]

(Huber 2000)
High-noise activity equation

Population activity

\[ A(t) = g(h(t)) \]

Membrane potential caused by input

\[ \tau \frac{d}{dt} h(t) = - h(t) + R I(t) \]

\[ \tau \frac{d}{dt} h(t) = - h(t) + R I^{ext}(t) + g(h(t)) \]

Attention:
- valid for high noise only, else transients might be wrong
- valid for high noise only, else spontaneous oscillations may arise

Random Connectivity/Asynchronous State

- mean drive
  \[ I_0 = J_0 g A_0 + I_0^{ext} \]
- variance
  \[ \sigma^2 = \frac{J_0}{C} A_0 \]
- improved mean field
  \[ A_0 = h_0 \]

\[ v = \langle \sigma \rangle^{-1} \left[ \frac{1}{2} \left( I_0 - C_i + I_0^{ext} \right) \right] = g(h_0, \sigma) \]

Spatial coupling

Microscopic vs. Macroscopic Coupling

Cortical columns:
Orientation tuning
Detour: Receptive fields (see also lecture 4)

Detour: Receptive field development

Receptive fields:
Retina, LGN

Receptive fields:
visual cortex V1

Orientation selective

Detour: orientation selective receptive fields
Detour: Receptive fields

Neighboring cells in visual cortex
Have similar preferred orientation:
cortical orientation map

Detour: orientation selective receptive fields

Receptive fields:
visual cortex V1

Stimulus orientation

Continuous Networks

Course coding

Many cells respond to a single stimulus with different rate

Several populations → Continuum

Blackboard

Continuum: stationary profile

$A(\theta, t) = A(\theta)$

Preferred orientation (deg) θ

Accuracy [deg]
Head direction cells (and line attractor)

Hippocampal Place Cells
Main property: encoding the animal's location

Head-direction Cells
Main property: encoding the animal's heading

Head-direction Cells
Main property: encoding the animal's allocentric heading

Basic phenomenology

I: Bump formation

strong lateral connectivity
II. Edge enhancement

Weaker lateral connectivity

Continuum: stationary profile

\[ A(\theta, t) = A(\theta) \]

Comparison simulation of neurons and macroscopic field equations

Exercises 2, 3, 4 from 11:15 - 12:45

Exercise 1: homogeneous stationary solution

Exercise 2: bump solution

Exercise 3: bump formation

Blob forms where the input is

The end

Liquid state/
Information buffering

Ultra-short-term information buffering (‘echo’)

(Maass et al., 2002, H. Jäger 2004)

640 excitatory n.
160 inhibitory n.
20ms time constant
50 connections/n.
Ultra-short-term information buffering ('echo')

\[ I(t) \rightarrow A(t) \]

\[ \text{out}(t) = \alpha \sum g(t-t') \]

\[ = \alpha \int g(s) A(t-s) ds \]

microscopic readout
mean-field readout

Mayor & Gerstner