Lecture 5 – Networks of Neurons and **Associative Memory**

- Introduction
- Associative memory and Classification by similarity
- Detour: magnetic materials
- Associative Memory
- Hopfield Model
- Memory Capacity

**Wulfram Gerstner, EPFL**

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**Systems for computing and information processing**

<table>
<thead>
<tr>
<th>Brain</th>
<th>Computer</th>
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<tbody>
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<td></td>
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Distributed architecture

<table>
<thead>
<tr>
<th>1 CPU</th>
<th>(10^10 transistors)</th>
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Von Neumann architecture

<table>
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<tr>
<th>1 CPU</th>
<th>(10^10 transistors)</th>
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</table>

No separation of processing and memory

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**Associations, Associative Memory**

*Read this text NOW!*
- Classification by similarity: **pattern recognition**

<table>
<thead>
<tr>
<th>image</th>
<th>Prototypes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noisy image</td>
<td>A</td>
</tr>
</tbody>
</table>

**Blackboard:**

- recognize/understand images: **pattern recognition**

\[ |x - p^T_1| \leq |x - p^A_1| \]

<table>
<thead>
<tr>
<th>Noisy image</th>
<th>Prototypes</th>
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<tbody>
<tr>
<td>A</td>
<td>T</td>
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</table>

**Aim: Understand Associative Memory**

Pattern recognition/Pattern completion

<table>
<thead>
<tr>
<th>Noisy image</th>
<th>Full image</th>
</tr>
</thead>
<tbody>
<tr>
<td>Partial word</td>
<td>T</td>
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</tbody>
</table>

**Brain-style computation**

**Detour: magnetism**

- Noisy magnet

- pure magnet
Detour: magnetism

Elementary magnet

\[ S_i (t + 1) = \text{sgn} \sum_j w_{ij} S_j \]

Sum over all interactions with i

Blackboard: example

Anti-ferromagnet

\[ S_i (t + 1) = \text{sgn} \sum_j w_{ij} S_j \]

Sum over all interactions with i

Associative memory

Elementary pixel

\[ S_i (t + 1) = \text{sgn} \sum_j w_{ij} S_j \]

Sum over all interactions with i

Hopfield model

Exercise 1: Associative memory (1 pattern)

- Introduction
- Associative Memory and Classification by similarity
- Detour: magnetic materials
- Detour: magnetic materials
- Hopfield Model
- Dense networks (mean-field)

Next lecture at 10h15

9 neurons
- define appropriate weights
- what happens if one neuron wrong?
- what happens if n neurons wrong?

9 neurons
- define appropriate weights
- what happens if one neuron wrong?
- what happens if n neurons wrong?
Associative memory – many patterns

Prototype \( \mathbf{p}_1 \)

Prototype \( \mathbf{p}_2 \)

Hopfield model

\[ S_i(t + 1) = \text{sgn} \left( \sum_{j} w_{ij} S_j \right) \]

Interacting neurons

Prototype \( \mathbf{p}_1 \)

\( \text{Finds the closest prototype} \)
(\( \text{i.e. maximal overlap} \))

(\( \text{similarity} \)) \( m^H \)

Hopfield model

**Computation**
- without CPU,
- without explicit memory unit

**Where do the connections come from?**

**Hebbian Learning**

When an axon of cell \( j \) repeatedly or persistently takes part in firing cell \( i \), then \( j \)'s efficiency as one of the cells firing \( i \) is increased

\( \text{Hebb, 1949} \)

- local rule
- simultaneously active (correlations)

**Hebbian Learning**

**DEM0**

Random patterns, fully connected:

Hopfield model

\[ S_i(t + 1) = \text{sgn} \left( \sum_{j} w_{ij} S_j \right) \]

\( \text{This rule is very good for random patterns} \)

\( \text{It does not work well for correlated patterns} \)
**Hebbian Learning – Associative Recall**

Recall:
Partial info

Item recalled

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**Associative Recall**

Tell me the color
the following list of 5 items:

Red
Blue
Yellow
Green

**Stroop effect:**

Slow response: hard to work
Against natural associations

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**Exercises 2+3 now: learning of prototypes**

Prototype $\hat{p}^1$
Prototype $\hat{p}^2$

be as fast as possible:

Sum over all interactions with $i$

a) Show that (1) corresponds to a rate learning rule

$$\frac{d}{dt} w_{ij} = \alpha_i \nu_j^{pre} (\gamma_j + \delta) \nu_j^{post} - \delta$$

Assume that weights are zero at the beginning.

Each pattern is presented (enforced) during 0.5 sec (One after the other).

b) Compare with:

$$\frac{d}{dt} w_{ij} = \alpha_i + \alpha_j^{pre} \nu_j^{pre} + \alpha_j^{post} \nu_j^{post} + \alpha_j^{post} \nu_j^{pre} \nu_j^{post} + \ldots$$

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**Exercise 2+3 (start now, rest homework)**

Assume 4 patterns. At time $t=0$, overlap with Pattern 3, no overlap with other patterns.

discuss temporal evolution

(assume that patterns are orthogonal)

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**Associative Recall**

Hierarchical organization of Associative memory

animals

birds

fish

Name as fast as possible

an example of a bird

swan (or goose or raven or …)

Write down first letter: $s$ for swan or $r$ for raven …
Associative Recall
Nommez au plus vite possible un exemple d’un /d’une
name as fast as possible an example of a
outil tool
couleur color
fruit fruit
instrument music instrument

Lecture 5 – Network of neurons and associative memory

- Introduction
- Classification by similarity
- Detour: magnetic materials
- Associative Memory
- Hopfield model
- How many patterns?

Memory Capacity

Q: How many prototypes can be stored?

Prototype \( p^1 \)
Prototype \( p^2 \)

Q: How many prototypes can be stored?

Prototype \( p^1 \)
Prototype \( p^2 \)

Dynamics (2)
\[ S_i (t + 1) = \text{sgn} \sum_j w_{ij} S_j \]
Sum over all interactions with i

Exercise 4 now: Associative memory

Q: How many prototypes can be stored?

Prototype \( p^1 \)
Prototype \( p^2 \)

End of lecture, exercise+
Computer exercise: 12:00

Random patterns
Interactions (1)
\[ w_{ij} = \frac{1}{N} \sum_{\mu} p_i^\mu p_j^\mu \]
Sum over all prototypes

Random patterns \( \rightarrow \) random walk

a) show relation to erf function: importance of \( p/N \)
b) network of 1000 neurons – allow at most 1 wrong pixel?
c) network of \( N \) neurons – at most 1 promille wrong pixels?

The end