Noisy neurons and coding (again):
variance, autocorrelation, stochastic resonance

Lecture 9 of course:
Neural Networks and Biological Modeling

- Noise in the brain
- Variability=noise?
- Noise Models
- Stochastic spike arrival (diffusive noise)
- Comparison of noise models
- Coding and stochastic resonance

BOOK: Spiking Neuron Models, W. Gerstner and W. Kistler
Cambridge University Press, 2002

Chapter 5

Variability of spike trains = noise?

- Variability of interspike intervals

Despite constant external input,
broad interspike interval distribution

Variability of spike trains across repetitions

- Variability across repetitions

K repitions

\[ PSTH(t) = \frac{n(t, t + \Delta t)}{K \Delta t} \]

K=500 trials

Peri-Stimulus Time Histogram

Signal: action potential (spike)

10,000 neurons
3 km wires

Input to a single neuron

Consider 1 neuron

Variability of interspike intervals

u [mV]

ISI [ms]

t [ms]

Variability of interspike intervals across repetitions

stim
Variability of spike trains = noise?

**Sources of noise**

- Intrinsic noise (ion channels)
- Network noise (background activity)

**Intrinsic noise: Channel noise**

- Na+ channel from rat heart.
- A 10 sequential traces from a patch containing 4 or 5 channels.
- Bottom: average gives current time course.

**B: Opening times of single channel events**

**Temporally input shows reliable neurons**

Intrinsic reliability of neurons


d 3 repetitions

Exp. Data:
A. Rauch, H. Lüscher, U. Bente

See also: Mainen & Sejnowski

Fluctuating input signal → Reliable spike timing

Variability of spike trains = noise?

**Sources of noise**

- Intrinsic noise (ion channels)
- Network noise (background activity)

**Noise models**

**Relation between the two models:**

Review from lecture 8

- Noise in the brain
- Variability = noise?
- Noise in Integrate-and-fire
  - mathematical noise models

**Noise in the brain**

**Variability = noise?**

important

Less important

**Review from lecture 8**

- Noise in the brain
- Variability = noise?
- Noise in Integrate-and-fire
  - mathematical noise models

**Noise models**

Relation between the two models:

later today

now
Lecture 9 – noisy neurons and coding again: autocorrelation, variance, stochastic resonance

- Noise in the brain
- Variability = noise?
- Noise Models
  - stochastic spike arrival (passive membrane)
  - Comparison of noise models
  - Coding and stochastic resonance

Review of lecture 8

Input to a single neuron:
- stochastic spike arrival

Consider 1 neuron

Assumption of Stochastic spike arrival:
in network of exc. neurons, total spike arrival rate $A(t)$

$$R(t) = R(t) + I_{\text{noise}}$$

Expected voltage at time $t$ $$\langle u(t) \rangle = ?$$

Variance of voltage at time $t$

$$\langle \Delta u(t) \Delta u(t) \rangle = \langle u(t)u(t) \rangle - \langle u(t) \rangle^2 =$$

Mean current in passive membrane

Exercise 1 now: Diffusive noise (stochastic spike arrival)

1. Assume that for $t>0$ spikes arrive stochastically with rate $R(t) = \dot{u}(a - u_{\text{rest}})$
   - Calculate mean voltage

2. Assume autocorrelation
   - Calculate $\langle u(t)u(t) \rangle = ?$

Next lecture: 10h20
Indicative evaluation of Bachelor/Master courses given to EPFL students

from Monday, April 18 (TODAY, NOW!!!!) to Sunday, May 8 at midnight

Summary:
All the courses of all the sections are evaluated.
A single question on the course in general.

Stochastic spike arrival – diffusive noise

Part 2 of Lecture 9 of course:
Noisy neurons and coding (again):
autocorrelation, variance, stochastic resonance

Neural Networks and Biological Modeling
-review of lecture 8
-stochastic spike arrival (diffusive noise)
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Chapter 5

Diffusive noise (stochastic spike arrival)

Stochastic spike arrival:
excitation, total rate $R_e$
inhibition, total rate $R_i$

Passive membrane (no threshold)

$$ \tau \frac{d u}{dt} = -(u - u_{rest}) + \sum_{j=1}^{N} \frac{Q_j}{C} \delta(t-t'_j) $$

$\tau$ current pulses

EPSC

Diffusive noise (stochastic spike arrival)

Stochastic spike arrival:
excitation, total rate $R_e$
inhibition, total rate $R_i$

Syncaptic current pulses

$$ \tau \frac{d u}{dt} = -(u - u_{rest}) + \sum \frac{Q_j}{C} \delta(t-t'_j) + \sum \frac{Q_j}{C} \delta(t-t'_j') $$

EPSC IPSC

$\tau$ current pulses

Justify autocorrelation of spike input:

Poisson process in discrete time

In each small time step $\Delta t$

Prob. of firing $p = \nu / \Delta t$

Firing independent between one time step and the next

Stochastic spike arrival:

Current spikes
Exercise: Poisson process in discrete time

Stochastic spike arrival: excitation, total rate

In each small time step $\Delta t$
Prob. Of firing $p = \nu \Delta t$

Firing independent between one time step and the next

Show that correlation for

$$\langle S(t)S(t') \rangle = e^t \theta(t-t') + \nu^2$$

Next lecture: 11h15

Comparison of Noise Models and Stochastic Resonance

Part 3 of Lecture 9 of course:
Noisy neurons and coding (again):
- autocorrelation, variance, stochastic resonance
- Comparison of noise models
- Coding and stochastic resonance

Neural Networks and Biological Modeling
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Chapter 5

Diffusive noise (stochastic spike arrival)

Membrane potential density: Gaussian

Superthreshold vs. Subthreshold regime

Neural Networks and Biological Modeling
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Chapter 5
Lecture 9 – noisy neurons and coding again: autocorrelation, variance, stochastic resonance

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Noise models: from diffusive noise to escape rates

\[ \rho(t) = f(u(t), u'_0(t)) \propto \exp\left(-\frac{(u(t) - \bar{g})^2}{2\sigma^2}\right) \left[1 + u'_0(t)\right] \]

Comparison: diffusive noise vs. escape rates

\[ P(t|t') = \frac{\rho(t') \exp\left(-\int \rho(t') dt'\right)}{\text{escape rate}} \]

\[ P(t|t') = \begin{cases} \rho(t') & \text{Gaussian about } t' \\ \text{escape rate} & \text{survivor function} \end{cases} \]

\[ P(t|t') = \begin{cases} \rho(t') & \text{Gaussian about } t' \\ \text{escape rate} & \text{survivor function} \end{cases} \]

Stochastic Resonance: changing the noise level

\[ I(t) = I_0 \cos(\omega t) \]

\[ I_n(t) = \sigma^2(t) \]

\[ u \]

\[ \bar{g} \]

\[ \text{Sinusoidal input rates} \]

\[ \text{Plus noise} \]

\[ \text{Plus threshold} \]
Stochastic Resonance: changing the noise level

The problem of neural coding:
Temporal codes

Transmission of periodic signal

Exercise 3 now: Coding by time to first spike

With deterministic model
With Poisson model
With noisy IF (escape noise)

Next lecture: Question session
Miniproject 12h00

The end